Optimal Force–Velocity Profile in Ballistic Movements–*Altius: Citius or Fortius?*

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ABSTRACT

SAMOZINO, P., E. REJC, P. E. DI PRAMPERO, A. BELLI, and J.-B. MORIN. Optimal Force-Velocity Profile in Ballistic Movements-Altius: Citius or Fortius? Med. Sci. Sports Exerc., Vol. 44, No. 2, pp. 313-322, 2012. Purpose: The study's purpose was to determine the respective influences of the maximal power (P_{max}) and the force-velocity ($F_{-\nu}$) mechanical profile of the lower limb neuromuscular system on performance in ballistic movements. Methods: A theoretical integrative approach was proposed to express ballistic performance as a mathematical function of \overline{P}_{max} and F-v profile. This equation was (i) validated from experimental data obtained on 14 subjects during lower limb ballistic inclined push-offs and (ii) simulated to quantify the respective influence of \overline{P}_{max} and F-v profile on performance. **Results**: The bias between performances predicted and obtained from experimental measurements was 4%-7%, confirming the validity of the proposed theoretical approach. Simulations showed that ballistic performance was mostly influenced not only by \overline{P}_{max} but also by the balance between force and velocity capabilities as described by the $F_{-\nu}$ profile. For each individual, there is an optimal F-v profile that maximizes performance, whereas unfavorable F-v balances lead to differences in performance up to 30% for a given \bar{P}_{max} . This optimal $F-\nu$ profile, which can be accurately determined, depends on some individual characteristics (limb extension range, \overline{P}_{max}) and on the afterload involved in the movement (inertia, inclination). The lower the afterload, the more the optimal $F_{-\nu}$ profile is oriented toward velocity capabilities and the greater the limitation of performance imposed by the maximal velocity of lower limb extension. Conclusions: High ballistic performances are determined by both maximization of the power output capabilities and optimization of the $F-\nu$ mechanical profile of the lower limb neuromuscular system. Key Words: MAXIMAL POWER, JUMPING PERFORMANCE, LOWER EXTREMITY EXTENSION, INCLINED PUSH-OFF, EXPLOSIVE STRENGTH, MUSCLE FUNCTION

Balistic movements, notably jumping, have often been investigated to better understand the mechanical limits of skeletal muscle function *in vivo*, be it in animals (19,23) or in humans (6,9,20). One of the main questions scientists, coaches, or athletes ask when exploring factors for optimizing ballistic performance is which mechanical quality of the neuromuscular system is more important: "force" or "velocity" mechanical capability?

Ballistic movements may be defined as maximal movements aiming to accelerate a moving mass as much as possible, that is, to reach the highest possible velocity in the

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MEDICINE & SCIENCE IN SPORTS & EXERCISE_® Copyright © 2012 by the American College of Sports Medicine DOI: 10.1249/MSS.0b013e31822d757a shortest time during a push-off. From Newton's second law of motion, the velocity reached by the body center of mass (CM) at the end of a push-off (or takeoff velocity, v_{TO}) directly depends on the mechanical impulse developed in the movement direction (22,26,42). Because the ability to develop a high impulse cannot be considered as a mechanical property of the neuromuscular system, the issue is to identify which mechanical capabilities of the lower limbs determine the impulse. Developing a high impulse during a lower limb push-off and, in turn, accelerating body mass as much as possible have often been assumed to depend on power capabilities of the neuromuscular system involved in the movement (14, 19, 26, 29, 36, 40, 43). This explains the wide interest of sports performance practitioners in improving muscular power (9,10,12,14,27). On this basis, maximal power output (\overline{P}_{max}) may be improved by increasing the ability to develop high levels of force at low velocities (force capabilities or strength) and/or lower levels of force at high velocities (velocity capabilities) (10,11,27). The best strategy continues to be an everlasting source of interest and debate (5,10-12,14,29).

The overall dynamic mechanical capabilities of the lower limb neuromuscular system have been well described by inverse linear force–velocity (F-v) and parabolic power– velocity (P-v) relationships during various types of multijoint

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concentric extension movements (3,33,35,40,43). These relationships describe the changes in external force generation and power output with increasing movement velocity and may be summarized through three typical variables: the theoretical maximal force at null velocity (\overline{F}_0), the \overline{P}_{max} the lower limbs can produce over one extension, and the theoretical maximal velocity at which lower limbs can extend during one extension under zero load (\bar{v}_0) . These three parameters represent the maximal mechanical capabilities of lower limbs to generate external force, power output, and extension velocity, respectively. Because they characterize the mechanical limits of the entire neuromuscular function, they encompass individual muscle mechanical properties (e.g., intrinsic F-v and length-tension relationships, rate of force development), some morphological factors (e.g., crosssectional area, fascicle length, pennation angle, tendon properties), and neural mechanisms (e.g., motor unit recruitment, firing frequency, motor unit synchronization, intermuscular coordination) (9). Graphically, \overline{F}_0 and \overline{v}_0 correspond to the force axis and velocity axis intercepts of the linear F-v curve, respectively, and \overline{P}_{max} corresponds to the apex of the parabolic P-v relationship. Under these conditions, the relationship among these three parameters can be described by the following mathematical equation (41):

$$\overline{P}_{\max} = \frac{\overline{F}_0 \,\overline{v}_0}{4} \tag{1}$$

Consequently, two athletes with similar \overline{P}_{max} could theoretically present different F-v mechanical profiles, i.e., different combinations of \overline{F}_0 and \overline{v}_0 . The issue is therefore to determine whether the F-v profile may influence ballistic performances independently of \overline{P}_{max} . In other words, is it preferable to be "strong" or "fast" to reach the highest performance in ballistic movements? Such an analysis might provide greater insight into the relationship between mechanical properties of the neuromuscular system and functional performance, either to further explore animal motor behaviors (19,20) or to program athletic training in humans, as underlined in recent reviews (10,12,14).

The effects of the F-v mechanical profile on ballistic performance have been experimentally approached only through studies led in athletes with different training backgrounds (40,43), through different training protocols (5,7,11,16,27), or both (4,8). However, in these studies, the various F-vprofiles of athletes were also associated with various \overline{P}_{max} values among subjects, making it impossible to identify the sole effect of the F-v profile. The influence of force and velocity capabilities on jumping performance has been recently addressed through a theoretical integrative approach mathematically expressing the maximal jump height an individual can reach as a function of \overline{F}_0 and \overline{v}_0 (37). However, the observed positive effects of \overline{F}_0 and \overline{v}_0 on performance were not independent from possible effects of \overline{P}_{max} , the latter being overlooked.

On the basis of this theoretical approach, the main aim of this study was to determine the respective influences of \overline{P}_{max} and F-v profile on performance in ballistic lower limb movements. Moreover, force and velocity contributions to power output depend on the load involved (10,14). Consequently, the secondary aim of this study was to investigate whether the effects of the F-v profile on ballistic performances (if any) depend on the afterloads (additional loads and/or push-off orientation against gravity) involved in the movement. To achieve these aims, the aforementioned theoretical analysis was compared with experimental measurements during jumping.

THEORETICAL BACKGROUND

This section is devoted to an analysis of ballistic performance through maximal jumps at different push-off angles. The entire lower limb neuromuscular system is considered as a force generator characterized by an inverse linear F-v relationship and a given range of motion. The maximal jumping performance can be well represented by the maximal v_{TO} ($v_{TO_{max}}$) of the body CM. As detailed in the recent theoretical integrative approach, jumping performance can be expressed as a function of some mechanical characteristics of lower limbs. In this approach mentioned above (see Samozino et al. [37]), $v_{TO_{max}}$ can be expressed as follows:

$$v_{\rm TO_{max}} = h_{\rm PO}\left(\sqrt{\frac{\overline{F}_0^2}{4\overline{v}_0^2} + \frac{2}{h_{\rm PO}}(\overline{F}_0 - g\sin\alpha)} - \frac{\overline{F}_0}{2\overline{v}_0}\right)$$
[2]

where g is the gravitational acceleration (9.81 ms⁻²), α is the push-off angle with respect to the horizontal (°), h_{PO} is the distance covered by the CM during push-off corresponding to the extension range of lower limbs (m), and \overline{F}_0 (N·kg⁻¹ of moving mass) and \overline{v}_0 (m·s⁻¹) are the maximal force at theoretical null velocity and the theoretical maximal unloaded velocity of lower limbs, respectively. The push-off angle α , assumed to be the same as the axis of the force developed, is considered constant over the entire push-off. In equation 2, the afterload opposing in motion is taken into account through inertia (i.e., the moving mass present here in the normalization of \overline{F}_0) and gravity ($g \times \sin \alpha$, i.e., the component of the gravity opposed to the movement).

The $F_{-\nu}$ mechanical profile of lower limbs can be represented by the ratio between \overline{F}_0 and $\overline{\nu}_0$, i.e., by the slope of the linear $F_{-\nu}$ relationship $(S_{F\nu})$ given by the following equation:

$$S_{Fv} = -\frac{\overline{F_0}}{\overline{v}_0} \tag{3}$$

(with the force graphically represented on the vertical axis of the F-v relationship).

Thus, the lower the S_{Fv} , the steeper the F-v relationship and the higher the force capabilities compared with velocity ones (7). Note that S_{Fv} and \overline{P}_{max} are theorized to be independent.

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Substituting equation 3 in equation 2 gives the following:

$$v_{\rm TO_{max}} = h_{\rm PO}\left(\sqrt{\frac{S_{F\nu}^2}{4} + \frac{2}{h_{\rm PO}}(\bar{F}_0 - g\sin\alpha)} + \frac{S_{F\nu}}{2}\right)$$
[4]

On the other hand, from equations 1 and 3, \overline{F}_0 can be expressed as a function of \overline{P}_{max} and S_{Fv} :

$$\overline{F}_0 = \sqrt{-4\overline{P}_{\max}S_{Fv}}$$
^[5]

Substituting equation 5 in equation 4 gives the following:

$$v_{\rm TO_{max}} = h_{\rm PO}\left(\sqrt{\frac{S_{F\nu}^2}{4} + \frac{2}{h_{\rm PO}}(2\sqrt{-\overline{P}_{\rm max}S_{F\nu}} - g \sin\alpha)} + \frac{S_{F\nu}}{2}\right) \qquad [6]$$

Consequently, $v_{TO_{max}}$ can also be expressed as a function of \overline{P}_{max} , S_{Fv} , and h_{PO} . Equation 6 is true for $h_{PO} > 0$, $\overline{P}_{max} > g \sin \alpha^2 / -4 S_{Fv}$, and $S_{Fv} < -g \sin \alpha^2 / 4 \overline{P}_{max}$ (see appendices, Supplemental Digital Content 1a, http://links.lww.com/MSS/A114, for details on the computations of these values). In the present study, equation 6 was (i) validated from experimental measurements and (ii) simulated to analyze the respective influences of \overline{P}_{max} and S_{Fv} on jumping performance.

METHODS USED IN THE EXPERIMENTAL VALIDATION

Subjects and experimental protocol. Fourteen subjects (age = 26.3 ± 4.5 yr, body mass = 83.9 ± 18.3 kg, stature = 1.81 ± 0.07 m) gave their written informed consent to participate in this study, which was approved by the local ethical committee and in agreement with the Declaration of Helsinki. All subjects practiced physical activities including explosive efforts (e.g., basketball, rugby, soccer); eight of them were rugby players (four played in the Italian first league). After a 10-min warm-up and a brief familiarization with the laboratory equipment, each subject performed two series of maximal lower limb push-offs: (i) horizontal extensions with different resistive forces allowing us to determine F-v relationships of the lower limbs and (ii) inclined jumps used to compare experimental performances with theoretical predictions.

Tests were realized on the Explosive Ergometer (EXER, see Figure, Supplemental Digital Content 2, http://links.lww.com/MSS/A115, for a schematic view of the EXER) consisting of a metal frame supporting one rail on which a seat, fixed on a carriage, was free to move (for more details, see Rejc et al. [34]). The total moving mass (seat + carriage) was 31.6 kg. The main frame could be inclined up to a maximum angle of 30° with respect to the horizontal. The subject could therefore accelerate himself or herself and the carriage seat backward by pushing on two force plates (LAUMAS PA 300; Parma, Italy) positioned perpendicular to the rail, the output of which was independent of the point of application of the force within a wide area. The velocity of the carriage seat along the direction of motion was continuously recorded by a wire tachometer (LIKA SGI,

Vicenza, Italy) mounted on the back of the main frame. Force and velocity analog outputs were sampled at a frequency of 1000 Hz using a data acquisition system (MP100; BIOPAC Systems, Inc., Goleta, CA). The instantaneous power was calculated from the product of instantaneous force and velocity values. Data were processed using the AcqKnowledge software (BIOPAC Systems, Inc.). An electric motor, positioned in front of the carriage seat, allowed us to impose known braking forces, acting along the direction of motion. The motor, controlled by a personal computer, was linked to the seat by a chain, its braking action initiating immediately at the onset of the subject's push. The braking force of the motor, ranging from about 200 to 2300 N, was set using a custombuilt LabVIEW program (National Instruments, Austin, TX).

For each test, the subject was seated on the carriage seat, secured by a safety belt tightened around the shoulders and abdomen, with the arms on handlebars. The starting position, set with feet on the force plates and knees flexed at 90°, was fixed thanks to adjustable blocks positioned on the rail of the EXER to prevent the downward movement of the carriage seat and, in turn, any countermovement.

F-v relationships of lower limb neuromuscular **system.** To determine individual F-v relationships, each subject performed horizontal maximal lower limb extension against seven randomized motor braking forces: 0%, 40%, 80%, 120%, 160%, 200%, and 240% of the subject's body weight. The condition without braking force (0% of body weight) was performed with the motor chain disconnected from the carriage seat. For each trial, subjects were asked to extend their lower limbs as fast as possible. Two trials, separated by 2 min of recovery, were completed at each braking force. Mean force (\overline{F}) , velocity (\overline{v}) , and power (\overline{P}) for the best trial of each condition were determined from the averages of instantaneous values over the entire push-off phase. The push-off began when the velocity signal increased and ended when the force signal (if takeoff) or the velocity signal (if no takeoff) fell to zero. As previously suggested (3,33,43), F-v relationships were determined by least squares linear regressions. Because P-v relationships are derived from the product of force and velocity, they were logically described by second-degree polynomial functions. *F*–*v* curves were extrapolated to obtain \overline{F}_0 (then normalized to total moving mass, i.e., body + carriage seat mass) and \bar{v}_0 , which correspond to the intercepts of the F-v curve with the force and velocity axis, respectively. According to equation 3, S_{Fv} was then computed from \overline{F}_0 and \overline{v}_0 . Values of \overline{P}_{max} (normalized to body + carriage seat mass) were determined from the first mathematical derivation of P-v regression equations. Moreover, to test the validity of equation 1, \overline{P}_{max} was also computed from this equation $(P_{\text{max}_{\text{TH}}})$.

Inclined push-off performance. To validate equation 6, each subject then performed two inclined maximal push-offs at three sled angles (α) (10°, 20°, and 30° above the horizontal) with the motor chain disconnected from the carriage seat, following the same procedures described above. v_{TO} was determined for each trial as the instantaneous

velocity value when the force signal fell to zero. Push-off distance (h_{PO}) was determined for each subject by integrating the velocity signal over time during the push-off phase.

Statistical analyses. All data are presented as mean ± SD. For each subject and each sled angle condition, the highest v_{TO} reached in the two trials was compared with $v_{\rm TO_{max}}$ computed according to equation 6, from $\bar{P}_{\rm max}$, $h_{\rm PO}$, and S_{Fv} . After checking distributions normality with the Shapiro–Wilk test, the difference between v_{TO} and $v_{TO_{max}}$ (bias) was computed and tested using a *t*-test for paired samples. To complete this comparison, the absolute difference between v_{TO} and $v_{TO_{max}}$ (absolute bias) was also calculated as $|(v_{TO_{max}} - v_{TO})v_{TO}^{-1}|100$ (36). Using the same comparison method, experimental values of \overline{P}_{max} were compared with theoretical values $(P_{\text{max}_{\text{TH}}})$. After checking the homogeneity of variances, the effect of sled angle was tested with a one-way ANOVA for repeated measures on $v_{\rm TO}$ and $v_{\rm TO_{max}}$. When a significant effect was detected, a post hoc Newman-Keuls comparison was used to locate the significant differences. For all statistical analyses, a P value of 0.05 was accepted as the level of significance.

METHODS USED IN THE SIMULATION STUDY

The relative influences of \overline{P}_{max} and S_{Fv} on $v_{TO_{max}}$ were analyzed via equation 6. First, $v_{TO_{max}}$ changes with S_{Fv} were determined for different \overline{P}_{max} values at different push-off angles (α). The range of \overline{P}_{max} and S_{Fv} values used in the simulations was obtained from data (\overline{P}_{max} , \overline{F}_0 , \overline{v}_0) previously reported for human maximal lower limb extensions: \overline{P}_{max} from 10 to 40 W·kg⁻¹ and S_{Fv} until to -40 N·s·m⁻¹·kg⁻¹ (32,33,36,43). The effect of h_{PO} on performance, previously studied and discussed (see Samozino et al. [37]), was not specifically treated here; h_{PO} was set at 0.4 m, which is a typical value for humans. Then, sensitivity analyses were performed to assess the respective weight of each variable plotting relative variations in v_{TOmax} against relative variations

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in \overline{P}_{max} and S_{Fv} at different push-off angles (α), each variable being studied separately.

RESULTS

Validation of the theoretical approach. Individual F-v and P-v relationships were well fitted by linear $(r^2 = 0.75 - 0.99, P \le 0.012)$ and second-degree polynomial $(r^2 = 0.70 - 1.00, P \le 0.024)$ regressions, respectively. Figure 1 shows these relationships for two typical subjects with different F-v profiles (i.e., different \overline{F}_0 , \overline{v}_0 , and S_{Fv}) and different $\overline{P}_{\text{max}}$ capabilities. Mean ± SD values of h_{PO} , \overline{v}_0 , \overline{F}_0 , $\overline{P}_{\text{max}}$, and S_{Fv} were 0.39 ± 0.04 m, 2.78 ± 0.63 m·s⁻¹, 24.2 ± 2.97 N·kg⁻¹ (or 17.3 ± 1.60 N·kg⁻¹ when normalized to body + carriage seat mass), $16.34 \pm 2.26 \text{ W} \text{kg}^{-1}$ (or $11.78 \pm$ 1.80 W kg⁻¹ when normalized to body + carriage seat mass), and -9.33 ± 3.31 N s m⁻¹ kg⁻¹ (or $-6.64 \pm$ 2.12 $N \cdot s \cdot m^{-1} \cdot kg^{-1}$ when normalized to body + carriage seat mass), respectively. The difference between \overline{P}_{max} and $P_{max_{TH}}$ was not significant and very low (absolute bias = $1.81\% \pm$ 0.76%), which shows the validity of equation 1. Mean \pm SD values of v_{TO} and $v_{TO_{max}}$, as well as mean values of absolute bias, are presented in Table 1. For each push-off angle, v_{TO} and $v_{TO_{max}}$ were not significantly different, and bias was $-0.05 \pm 0.17 \text{ m} \cdot \text{s}^{-1}$ (see Figure, Supplemental Digital Content 3, http://links.lww.com/MSS/A116, which shows bias and limits of agreement in a Bland-Altman plot). On the other hand, the effect of push-off angle was significant on both v_{TO} and $v_{TO_{max}}$, with differences between every condition (Table 1).

Theoretical simulations. As expected, \overline{P}_{max} positively affects $v_{TO_{max}}$, which is clearly shown in Figure 2 for both vertical ($\alpha = 90^{\circ}$) and horizontal ($\alpha = 0^{\circ}$) push-offs. The main original result was the curvilinear changes in $v_{TO_{max}}$ with S_{Fv} for a given \overline{P}_{max} (Fig. 2). Such variations highlight the existence of an optimal S_{Fv} ($S_{Fv_{opt}}$) maximizing $v_{TO_{max}}$ for given \overline{P}_{\max} and h_{PO} . Moreover, $S_{Fv_{opt}}$ values seem to change slightly as a function of both \overline{P}_{\max} and α values, ranging



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14

12

TABLE 1. Mean \pm SD of v_{TO} obtained with experimental and theoretical approaches, absolute bias between these two approaches, and *t*-test comparison results.

| α (°) | Experimental Values (<i>v</i> _{TO} (m·s ⁻¹)) | Theoretical Values (v _{TOmax} (m·s ⁻¹)) | <i>t</i> -Test | Absolute Bias (%) |
|-------|---|---|----------------|----------------------|
| 10 | 2.45 ± 0.22 | 2.43 ± 0.18 | ns | 4.40 ± 4.94 |
| 20 | 2.32 ± 0.25^{a} | 2.25 ± 0.16 ^a | ns | $6.56~\pm~5.46$ |
| 30 | 2.14 ± 0.23^{ab} | 2.07 ± 0.15^{ab} | ns | 5.73 ± 3.89 |

^a Significantly different from $\alpha = 10^{\circ}$

^b Significantly different from $\alpha = 20^{\circ}$

ns, nonsignificative difference between experimental and theoretical values.

from -18 to -6 N·s·m⁻¹·kg⁻¹ for the conditions simulated in Figure 2. The dependence of $S_{Fv_{opt}}$ on \overline{P}_{max} , α , and h_{PO} can be mathematically analyzed: the expression of $S_{Fv_{opt}}$ as a function of these three variables is a real solution canceling out the first mathematical derivative of $v_{TO_{max}}$ with respect to S_{Fv} (see appendices, Supplemental Digital Content 1b, http://links.lww.com/MSS/A114, for detailed computations of $S_{Fv_{opt}}$). Whatever the value of \overline{P}_{max} , $S_{Fv_{opt}}$ decreases when α increases (Fig. 3). For both vertical and horizontal pushoffs, the sensitivity analysis showed that $v_{TO_{max}}$ is more influenced by \overline{P}_{max} than by S_{Fv} , at least when the S_{Fv} reference value is equal to $S_{Fv_{opt}}$ (Fig. 4). Moreover, the respective effects of \overline{P}_{max} and S_{Fv} on $v_{TO_{max}}$ seem to decrease with decreasing α (Fig. 4).

DISCUSSION

The original and main findings of this study are that ballistic performance of the lower limbs depends on both \overline{P}_{max} capabilities and the F-v profile, with the existence of an individual optimal F-v profile corresponding to the best balance between force and velocity capabilities. This optimal F-v profile, which can be accurately determined, depends on some individual characteristics (limb extension range, \overline{P}_{max}) and on the afterload involved in the movement



FIGURE 3—Changes in optimal $F-\nu$ profile (S_{Fvopt}) as a function of the push-off angle (α) for different \overline{P}_{max} values. The h_{PO} is fixed here at 0.4 m.

(inertia, inclination). The concept of optimal F-v profile and the proposed approach make it possible to clarify some scientific issues previously discussed about the mechanical capabilities of lower limbs that determine ballistic performance and about the relationships between lower limb neuromuscular system structure and function. The following discussion is devoted to detailing these different points.

Validity of the theoretical approach. These findings were obtained using a theoretical integrative approach based on fundamental principles of dynamics and on the F-v linear model characterizing the dynamic mechanical capabilities of the neuromuscular system during a lower limb extension. This linear model, as well as the parabolic P-v relationship, has been well supported and experimentally described for



FIGURE 2—Changes in maximal CM v_{TO} (v_{TOmax}) reached at the end of a lower limb push-off, as a function of the changes in the F-v profile (S_{Fv}) for different \overline{P}_{max} values and at two push-off angles (α). The h_{PO} is fixed here at 0.4 m. For the vertical push-off ($\alpha = 90^\circ$), the corresponding jump height (obtained from basic ballistic equations) is presented on the additional y axis. *Open circles* represent the v_{TOmax} reached for an optimal F-v profile ($S_{Fv_{wx}}$).



FIGURE 4—Sensitivity analyses: relative changes in maximal CM $v_{TO}(v_{TOmax})$ as a function of the relative variations of \overline{P}_{max} and F-v profile (S_{Fv}) for two α values. The reference value for \overline{P}_{max} is 25 W·kg⁻¹ and corresponds to the optimal F-v profile value for S_{Fv} (-14.0 for $\alpha = 90^{\circ}$ and -8.20 for $\alpha = 0^{\circ}$). The h_{PO} is fixed here at 0.4 m. For S_{Fv} , the higher the normalized variation, the lower the value because S_{Fv} values are only negative and the more the F-v profile tends toward force capabilities.

multijoint movements (3,33,43,44). The linearity of the F-vrelationship, usually presented as hyperbolic for isolated muscles (17), is explained by the integrative feature of the model. The force generator and, in turn, its maximal force (\overline{F}_0) , unloaded velocity (\overline{v}_0) , and power (\overline{P}_{max}) refer here to the entire in vivo neuromuscular system involving several muscles with different mixed fiber composition, architectural characteristics, anatomical joint configuration, level of neural activation, and specific coordination strategies (7-9,44). The limits of this theoretical approach have been previously discussed (37), but the significance and accuracy of its predictions have not been quantified yet. Besides validating equation 1 (\overline{P}_{max} and $P_{max_{TH}}$ are very close), the present results showed no differences between predicted $(v_{TO_{max}})$ and measured (v_{TO}) values, associated to a low absolute bias from 4% to 6.6%. This is within the range of reproducibility indices previously reported for different variables (performance, velocity, force, or power) measured during lower limb maximal extensions (3,18). These results support the validity of the proposed theoretical approach, which was strengthened by the sensibility of both predicted and experimental values to changes in push-off angles. Obviously, the accuracy of equation 6 is enhanced when muscular properties (\overline{P}_{max} and S_{Fv}) are assessed in the same conditions (e.g., joints and muscle groups involved, range of motion) under which the actual performance is studied, as it was done here on the EXER.

Muscular capabilities determining jumping performance. Among the muscular characteristics determining jumping performance, \overline{P}_{max} has the greatest weight. Although expected, the importance of \overline{P}_{max} in setting ballistic performance needed to be established, as concluded by Cronin and Sleivert (12) in their recent review: "power is only one aspect that affects performance and it is quite likely that other strength measures may be equally if not more important for determining the success of certain tasks." The present results clearly demonstrate this idea. On the other hand, the dependence of ballistic performances on muscular

power capability brings new insights into the recurrent debate about the role of "power" in impulsive performance, such as jumping (22,26,42). On the basis of Newton's second law of motion, some authors stated that jumping performance does not depend on the muscular capability to develop power but rather on the capability to develop a high impulse (26,42). Even if fundamental principles of dynamics directly relate mechanical impulse to v_{TO} (and in turn jumping performance), the capability to generate impulse does not represent an intrinsic mechanical property of the lower limb neuromuscular system, contrary to \overline{P}_{max} . It is important to differentiate mechanical outputs (e.g., external force, movement velocity, power output, impulse, mechanical work) from mechanical capabilities of lower limbs $(\overline{P}_{max}, \overline{v}_0, \overline{F}_0)$. On the one hand, mechanical outputs represent the mechanical entities that can be externally measured during a movement and are often used to characterize movement dynamics from a mechanical point of view. On the other hand, mechanical capabilities of lower limbs characterize the mechanical limits of the neuromuscular function and refer to the theoretical maximal values of some mechanical outputs that could be reached by an individual. The proposed theoretical approach demonstrates that the ability to develop a high impulse against the ground and, in turn, the ability to reach maximal CM velocity at the end of a push-off are highly related to the \overline{P}_{max} the lower limbs can produce (over a given extension range).

That said, the present results show that \overline{P}_{max} is not the only muscular property involved in jumping performance. Indeed, two individuals with the same \overline{P}_{max} (and the same h_{PO}) may achieve different performances, be it during a vertical jump or a horizontal push-off (Fig. 2). These differences are due to their respective $F-\nu$ profiles ($S_{F\nu}$), i.e., to their respective ratios between maximal force (\overline{F}_0) and velocity ($\overline{\nu}_0$) capabilities. For each individual (given his/her \overline{P}_{max} and h_{PO}), there is an optimal $F-\nu$ profile that maximizes performance. The more this $F-\nu$ profile differs from the optimal one, the lower the performance in comparison

with the one that could be reached with the same power capabilities (Fig. 2). The values of S_{Fy} observed here (from -16.8 to -4.9 N·s·m⁻¹·kg⁻¹) are consistent with \overline{F}_0 and \bar{v}_0 values previously reported (3,32,33,43). $\bar{P}_{\rm max}$ and \bar{F}_0 values were slightly lower than those reported during vertical push-offs (3,32,33), which is likely due to the specific sitting position imposed by the EXER compared with the totally extended hip configuration usually tested. Individuals, notably rugby players, as most of our subjects were, may present very different F-v profiles, as shown by coefficients of variation for S_{Fv} of beyond 30% compared with coefficients of variation below 20% for \overline{P}_{max} or \overline{F}_{0} . Most of these different individual F-v profiles differ from the optimal ones, thus characterizing unfavorable balances between force and velocity capabilities. Indeed, individual F-v profiles observed in this study ranged from 36% to 104% of the optimal ones maximizing vertical jumping performance. Simulations of equation 6 showed that such unfavorable F-v balances may be related to differences up to 30% in jump height between two individuals with similar power capabilities (Figs. 2 and 4). Consequently, we think that the F-v profile represents a muscular quality that has to be considered attentively not only by scientists working on muscle function during maximal efforts but also by coaches for training purposes.

Effect of afterloads on optimal F-v profile. The optimal F-v profile depends on some individual characteristics ($h_{\rm PO}$, $\overline{P}_{\rm max}$) and on the afterload opposing in motion (inertia, inclination). On the one hand, the F-v profile does affect jumping performances when S_{Fv} is expressed through values normalized to the total moving mass $(N \cdot s \cdot m^{-1} \cdot kg^{-1})$, which may be body mass, body mass plus additional loads, or projectile mass. Thus, the interpretation of F-v profiles is dependent on the movement considered. On the other hand, the computation of the optimal F-vprofile also takes account of the total moving mass: $S_{Fv_{out}}$ is a function of \overline{P}_{max} , itself expressed relative to moving mass. Consequently, for a given athlete, the optimal F-v profile is not the same for a javelin throw (high \overline{P}_{max} relative to moving mass) and for a shot put (low relative \overline{P}_{max} , see the different curves in Fig. 3). The optimal F-v profile also depends on the push-off angle and more generally on the magnitude of the gravity component opposing motion (the lower the push-off angle, the more the optimal F-v profile is oriented toward velocity capabilities). Thus, the optimal F-vprofile is not the same when seeking to maximize performance during the first push of a sprint or during a vertical jump; velocity capabilities are more important in the former case; force capabilities, in the latter. This is in line with the theoretical framework proposed by Minetti (28) showing that power output developed during maximal efforts is less dependent on muscle strength when the exercise does not involve gravity, as in horizontal extensions. Such horizontal (or very horizontally inclined) push-offs are thus especially limited by the velocity capabilities of lower limbs. The originality of the present theoretical approach is to allow the accurate determination of the optimal balance between

force and velocity capabilities (through $S_{Fv_{opt}}$) according to movement specificities. The subjects tested here presented an overall unfavorable balance toward velocity capabilities for vertical jumps (S_{Fv} from 36% to 104% of their respective $S_{Fv_{opt}}$) and toward force capabilities for horizontal push-offs (S_{Fv} from 66% to 227% of $S_{Fv_{opt}}$).

*F***–v** profile and athletic training. Assessing F-vprofiles when seeking to identify the optimal balance between force and velocity capabilities may be of interest to set training loads and regimens, as previously proposed using power–load relationships (10,20,27,38). Values of S_{Fv} allow comparisons among athletes independently from their power capabilities (which is not possible from only \overline{F}_0 and \overline{v}_0 values) and, thus, to know whether an athlete, as compared with another one, is characterized by a "force" or a "velocity" profile (Fig. 1). To the best of our knowledge, only Bosco (2) proposed an index to compare athletes' F-vprofiles dividing jump height reached with an additional load (100% of body mass) by unloaded jump height: the higher this index, the higher the force capabilities compared with the velocity ones. However, Bosco's index does not allow the orientation of training loads for a given athlete according to his/her own strengths and weaknesses and to movement specificities. Therefore, we propose the individual value of S_{Fv} , expressed relatively to $S_{Fv_{opt}}$, as a good and practical index to characterize the F-v profile and to design appropriate training programs. The present results showed that improving ballistic performance may be achieved through increasing power capabilities (i.e., shifting F-vrelationships upward and/or to the right [21]) and moving the F-v profile as close to the optimal one as possible. Such changes in the F-v relationship, notably in its slope, may be achieved by specific strength training (7,8,21). An athlete presenting an unfavorable F-v balance in favor of force (relatively to his/her optimal profile corresponding to target movement specificities) should improve his/her velocity capabilities as a priority by training with maximal efforts and light (e.g., <30% of one repetition maximum, the latter being close to \overline{F}_0) or negative loading, which is often called "ballistic" or "power" training (7,8,11,25,27). On the contrary, an athlete with an imbalanced F-v profile oriented toward velocity should follow a strength training with heavy loads (>75%-80% of one repetition maximum) to increase his/her force capabilities as a priority (7,8,27). In both cases, it is likely that (i) \overline{P}_{max} will increase and (ii) the *F*-*v* profile will be optimized (i.e., change toward the optimal one), partly or totally correcting unfavorable F-v balances. As shown in the present study, these two changes would both result in a higher performance. The mechanisms underlying these changes in F-v relationships, specific to the kind of training, include changes in mixed fiber composition, muscle architecture (hypertrophy, pennation angle), and neural activation (voluntary activation level, firing frequency, rate of EMG rise, intermuscular coordination strategies) (1,7,15,27). These theoretical findings support previous experimental results about the velocity (or

load)-specific changes in performance after training with light or heavy loads (10,25,27), with the additional originality of controlling the respective effects of F-v qualities and $\overline{P}_{\text{max}}$ capabilities.

F-v profile and optimal load. The proposed approach brings new insight into the understanding of the relationships between structure and mechanical function of the lower limb neuromuscular system and, notably, the effect of specific changes in the F-v relationship on athletic performance. The concept of the F-v profile could be related to the maximum dynamic output hypothesis proposed and discussed by Jaric and Markovic (20) and supported by recent studies (6,13,30). Their hypothesis states that the optimal load-maximizing power output in ballistic movements for physically active individuals corresponds to their own body weight and inertia (20). They argued that this optimal load would be related to the particular design of the muscular system (notably its mechanical properties), itself influenced by the actual load individuals regularly overcome during their daily activities. They pointed out, however, that the different evidences provided needed to be supported by theoretical frameworks describing the general aspects of the neuromuscular system's ability to provide the \overline{P}_{max} output against a particular load. This may be done using the theoretical approach proposed here. Indeed, the slope of the F-vrelationship and, thus, the ratio between \overline{F}_0 and \overline{v}_0 are directly related to the optimal velocity and force-maximizing power output and so to the corresponding optimal load. From F-v and P-v relationships (Fig. 1), the higher the \bar{v}_0 , the higher the optimal velocity and the lower the optimal load. Conversely, high \overline{F}_0 values are associated with high optimal loads. Consequently, the optimal load corresponds to the mass and inertia of the body only for individuals developing their individual $\overline{P}_{\rm max}$ during an unloaded vertical jump. Because (i) maximal jump height changes with F-vprofile for a given \overline{P}_{max} (Fig. 2, left panel) and (ii) jump height and power output (relative to body mass) developed during a vertical jump are positively related (36), the power

output developed during an unloaded maximal jump depends on the F-v profile. Consequently, jumping performance depends directly on the mean power output developed during push-off (for a given h_{PO}), and the latter can be maximized by both maximizing $\overline{P}_{\rm max}$ and optimizing the F-v profile. This is illustrated in Figure 5, which shows the power output developed during a vertical jump (expressed relatively to \overline{P}_{max}) according to the *F*-*v* profile expressed relatively to the optimal one (power output was computed from equations 6 and 9 of Samozino et al. (36), see appendices for more details, Supplemental Digital Content 1c, http://links.lww.com/MSS/A114). An optimal F-v profile, i.e., an optimal balance between \overline{F}_0 and \overline{v}_0 , allows the development of \overline{P}_{max} during an unloaded jump (Fig. 5, left panel) and thus maximization of jumping performance (Fig. 5, right panel). Consequently, the body mass represents the optimal load for individuals with optimal F-v profiles. An athlete with an unfavorable F-v balance develops a power output lower than $\bar{P}_{\rm max}$ during an unloaded jump. Such an athlete would produce \overline{P}_{max} against a load lower than body mass if he/she presents a velocity profile and higher than body mass in the case of a force profile. The present theoretical framework may help to explain and understand the possible interindividual differences in optimal load previously observed, discussed, and debated (6,20,24,30,31,39). The influence of training history recently proposed supports our findings because training background specificities directly affect the F-v profile (8,10,27), which influences the optimal load (39). This is in line with the maximum dynamic output hypothesis stating that strength-trained athletes (with high force capabilities) present optimal loads higher than their body mass (20,39). In animals and humans, the lower limbs' neuromuscular system is likely designed to work optimally against loads usually supported and mobilized (20,23). Consequently, animals would naturally present F-v profiles optimizing ballistic performance such as horizontal jumps, when these latter represent their main survival behavior.



FIGURE 5—Left panel: changes in power output developed during a vertical jump (expressed in $\%\bar{P}_{max}$) with changes in $F-\nu$ profile ($S_{F\nu}$, expressed in $\%\bar{S}_{Fvopt}$). Right panel: effect of the power output developed during a vertical jump (expressed in $\%\bar{P}_{max}$) on the jump height reached (expressed relatively to the jump height that could be reached, should the $F-\nu$ profile be optimal). Values of \bar{P}_{max} and h_{PO} were fixed here at 25 W·kg⁻¹ and 0.4 m, respectively.

CONCLUSIONS

Ballistic performance is mostly determined not only by the \overline{P}_{max} lower limbs can generate but also by the F-vmechanical profile characterizing the ratio between maximal force capabilities and maximal unloaded extension velocity. This F-v profile of lower limbs, independent from power capabilities, may be optimized to maximize performance. *Altius* is neither *citius* nor *fortius* but an optimal balance between the two. This optimal F-v profile depends on individual and movement specificities, notably on the afterload involved (inertia and gravity): the lower the afterload, the more the optimal F-v profile will be oriented toward velocity capabilities. Considering F-v profile may help better understand the relationships between neuromuscular system mechanical properties and functional performance, notably to optimize sport performance and training. This original me-

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chanical quality was put forward by a theoretical integrative approach and validated here from comparisons between theoretically predicted performances and experimental measurements during jumping. This approach was discussed here for lower limb extensions, but the results may be also applied to other multijoint muscular efforts, such as upper limb ballistic movements, or more complex movements such as sprint running.

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NOTATION

- CM body center of mass
- т body mass or moving mass (kg)
- gravitational acceleration on Earth (9.81 m·s⁻²) g \bar{F}
- mean external force developed over push-off along the push-off axis (relative to moving mass (N·kg⁻¹))
- \bar{v} mean CM velocity over push-off along the push-off axis (m·s⁻¹) P
- mean power output developed over push-off (relative to moving mass $(W kg^{-1}))$
- \overline{F}_0 theoretical maximal value of \bar{F} that lower limbs can produce during one extension at a theoretical null \bar{v} (relative to moving mass (N·kg⁻¹))
- theoretical maximal value of \bar{v} at which lower limbs can extend during one extension under the influence of muscle action in a theoretical unloaded condition (m s⁻¹)
- ${ar v_0 \over P_{
 m max}}$ maximal \bar{P} that lower limbs can produce during a push-off (W kg⁻¹)
- $P_{\max_{\text{TH}}}$ theoretical value of \overline{P}_{max} estimated from equation 1 (W·kg⁻¹)
- push-off distance determined by lower limb extension range (m) $h_{\rm PO}$
- $v_{\rm TO}$ CM velocity at takeoff $(m \cdot s^{-1})$
- maximal v_{TO} an individual can reach (m·s⁻¹) v_{TOmax}
- push-off angle with respect to the horizontal (°) α
- slope of linear F-v relationship (N·s·m⁻¹·kg⁻¹) S_{Fv}
- optimal value of S_{Fv} maximizing $v_{TO_{max}}$ for given values of \overline{P}_{max} and h_{PO} (N s m⁻¹ kg⁻¹) $S_{Fv_{opt}}$